

MICHAEL HALLETT

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Office Hours: To Be Announced.

Lectures: Officially (according to the timetable) Monday, Wednesday, 10.05–11.25: see below.

Summary of Material. The aim of this course is to examine some parts of the geometry developed by Euclid in his remarkable work *The Elements* (c. 300 BCE), and in particular the role played in it by the famous ‘parallel postulate’. *The Elements* was based on 23 definitions, some so-called ‘Common Notions’, and 5 postulates, all except one of these apparently innocuous. This exception was Postulate 5, the Parallel Postulate (‘EPP’ for short), and it is this postulate on which we will eventually focus. First, we will look at the nature of the system, in particular at all the things which Euclid explicitly states, and (trickier!) what he implicitly assumes, and the way all this is made use of in Book I, specifically through the proofs. After this, we will then look at the very important role the EPP plays in the development of Euclid’s system, why it is brought in, and what Euclid succeeds in proving with it. We will then turn (as Euclid never does in any obvious way) to look at the status of the postulate. In particular, we will focus on Proclus’s claim (in his commentary on Book I of Euclid’s *Elements* (fifth century of the CE), that the EPP should *not* be listed among the postulates, but rather should be a *proved theorem*. Proclus himself attempted a proof, which we will examine closely, since its unsatisfactoriness reveals how interconnected the EPP is with other assumptions. There were various attempts made later, particularly by the Islamic geometers, and we will look at some of these attempts, too. The most interesting of these is one made much later by the (decidedly non-Ancient) English mathematician John Wallis in the 17th c. The reason for this looking closely at this proof is largely for the light that it sheds on many of the implicit things in Euclid’s system which we will have stressed, and also for the way it reveals the intricacies of the EPP and the way it intersects with other parts and assumptions of Euclid’s system. Lastly, and we will sum up this quickly, the failure to provide convincing proofs of the EPP from the rest of Euclid’s system, particularly through attempts made by trying to prove it by *reductio* (assuming the negation of EPP, and then searching for a contradiction), eventually led to the consideration of what are now called *non-Euclidean geometries*. In particular, we will raise here the question of how this kind of enquiry led to an extension of Euclid’s project, particularly an extension to the fundamental question which occupied him of what rests on what.

The course will therefore have 5 distinct parts:

- Part I: Euclid’s system, and particularly what is implicitly assumed.
- Part II: The deductive role of the EPP.
- Part III: The status of the postulates, and in particular the EPP (Proclus).
- Part IV: Proofs of the EPP. particularly Wallis’s.
- Part V: Briefly, the birth of non-Euclidean geometries.

Course Material

1. The basis of the course will be the text of Euclid contained in Thomas Heath’s translation of Euclid in *The Thirteen Books of Euclid’s Elements*, in particular Volume 1, which contains Euclid’s Book I. Heath’s historical notes and commentaries are also very useful.

2. Also very important to us will be the text of Proclus's *Commentary on the First Book of Euclid's*

Marking and Assessment: The final mark is composed of reading assignments (20%), a short sketch paper due around the middle of the term (30%), and based on set questions, and a final paper (also on set questions) due in the exam period with deadline as set by the university's rules on 'take-home